Published in the Russian Federation
European Journal of Technology and Design
Has been issued since 2013.
ISSN: 2308-6505
E-ISSN: 2310-3450
Vol. 5, No. 3, pp. 108-122, 2014

DOI: 10.13187/ejtd.2014.5.108
www.ejournal4.com

UDC 699

Forecasting of a Short Life Baked Product Using Exponential Smoothing and Markov Method

[1] Bijesh Paul
[2] Jayadas. N.H.

[1-2] Division of Mechanical Engineering, School of Engineering,
Cochin University of Science and Technology (CUSAT), India
[1] Research scholar
[2] Dr., Associate Professor
E-mail: bijeshpaul@hotmail.com

Abstract

The objective of this paper is to develop a demand forecast model for a short life baked product. The initial forecast is obtained by using exponential smoothing and the error corresponding to each day is estimated for this forecast. A control chart is plotted for these errors after determining its upper control limit and lower control limit. A generalized Markov algorithm is applied to these errors and the demand of different states are determined. The demand corresponding to the state with maximum probability is taken as optimal demand. The obtained results can act as a basis for better planning of demand of short life baked products in India.

Keywords: Demand; Exponential smoothing; Forecasting; Markov algorithm; Random; State; Planning.

Introduction

Almost all organizations analyses past sales data and predict the future sales based on this past data. An attempt has been done to predict future sales based on the sales data of two successive months collected from a reputed firm. Various statistical techniques are available for forecasting. Nice properties of a weighted moving average would be one where the weights not only decrease as older and older data are used, but one where the differences between the weights are "smooth". Obviously the desire would be for the weight on the most recent data to be the largest. The weights should then get progressively smaller the more periods one considers into the past. The exponentially decreasing weights of the basic exponential smoothing forecast fit this bill nicely. The forecast equation is given by:

$F_{t+1} = \alpha D_t + (1 - \alpha) F_t$, Where α is a smoothing parameter between 0 and 1. Here we assume $\alpha = .2$.

This model is best suited to time-series data. Initially demand is predicted using exponential smoothing method and the error of demand is modelled by using Markov method. Markov based algorithm can be used to forecast in that environment where limited past data is available. Further random component of demand can be modelled by using Markov chain based forecasting model. This is because transition probabilities in Markov model represent the influence of all random factors. Hence we apply Markov based algorithm to the errors of forecast of the conventional model namely exponential smoothing.

Literature review

Bijesh and Jayadas formulated an algorithm for short life cycle supply chain based on Markov model (2013) [1]. The above algorithm gives us a useful and financially feasible technique to determine the demand forecast whenever the demand data given is randomly distributed. A Grey–Markov forecasting model has been developed by Huang, He and Cen in 2007. This paper was based on historical data of the electric-power requirement from 1985 to 2001 in China, and forecasted and analyzed the electric- power supply and demand in China [2]. In 2007, Akay and Atak have formulated a Grey prediction model with rolling mechanism for electricity demand forecasting of Turkey [3]. Boylan and Syntetos (2006) commented on the importance of capturing the combined forecasting stock control operation through metrics relating to 'service level' and inventory costs (accuracy implication metrics); i.e., by considering what is important from a practitioner's perspective [4]. Armstrong and Green (2005) aver that "exponential smoothing is the most popular and cost effective of the statistical extrapolation methods [5]. Timmermann and Granger (2004) highlighted the need to evaluate forecast results using utility functions. Often the predictive approach that is best based on a given accuracy metric will not be the one that outperforms competitors if utility measures are employed, such as financial outcomes, inventories, customer satisfaction, or socio-economic benefits [6]. Allen and Fildes (2001) review the literature on the advantages of using disaggregate data, one of which is the additional information available due to heterogeneity across individual markets. However, they also argue that the relative performances of aggregate and disaggregate approaches might depend on the specifics of the forecasting exercise. Categorical data sequences can be modelled by using Marko chains see for instance [8, 9 and 10]. The applications of grey model for energy forecasting problems have resulted in several research papers [7]. In 2001, Zhang and He have developed a Grey–Markov forecasting model for forecasting the total power requirement of agricultural machinery in Shangxi Province [11].

Estimation of parameters of model

The parameters of the model are the initial probability matrix P^0 and transition probability matrix P^{ij}. Count the no of occurrences of each state in the given month t of the observed data to determine the initial probability matrix or initial probability vector P^0. For any observed data sequence we can determine the transition probability matrix by counting the transitions from one particular state to all other possible states. Any subsequent matrix indicating the probability of a state at that time can be determined by using above two matrixes. Deduce the current probability vector for the succeeding months t+2, t+3 as

$P^1=P^0* TPM$

$P^2=P^1* TPM$

....

$P^m= P^{m-1} *TPM$

Methodlogy

1. Observed demand data for a short life cycle product is collected for any two successive months.
2. Apply exponential smoothing to the collected data and estimate the errors in forecasting for all the days of two successive months.
3. Errors are plotted on a control chart in the order that they occur. The centerline of the chart represents an error of zero. Note the two other lines, one above and one below the centerline. They are called the upper and lower control limits because they represent the upper and lower ends of the range of acceptable variation for the errors.

4. In order for the forecast errors to be judged "in control" (i.e., random), two things are necessary. One is that all errors are within the control limits. The other is that no patterns (e.g., trends, cycles, and no centered data) are present.
5. Implement the generalized algorithm based on Markov method for the errors of the forecasted model.
6. Deduce the initial probability matrix and the Transition probability matrix for the different states of errors of demand.
7. By utilizing the above two matrices the probability of different states of demand for any future period can be determined. The evolution of the system is determined by multiplying the transition matrix by the previous state vector (probability matrix), which is a stochastic vector representing the probabilities of the system being in any one of the given states
8. Choose the state with maximum probability from the obtained current probability vector
9. Determine the annual savings by adopting the demand of the state with maximum probability.

Algoritm for demand prediction based on the combined exponential smoothing and markov based analysis

1. Collect the observed data for sales of a particular product with minimum shelf life for any two consecutive or successive months, say t and t+1.
2. Apply exponential smoothing to the collected data and estimate the errors in forecasting for all the days of two successive months.
3. Determine the upper limit and lower limit of the errors in forecasting by exponential smoothing for the t^{th} month. Determine the range or band width of the error as the difference between upper limit and lower limit for the t^{th} month.
4. Discretize the obtained range into states or class intervals with minimum possible no of sample size. Let us denote these states as X_1, X_2, X_3...........X_n.
5. Determine the initial probability vector P^o for the month t. This matrix gives the initial probability of all states say X_1, X_2, X_3...........X_n in month t.

a. List out all the days (m) in a month in the month t as the first column, in the ascending order of the table.

b. In second column enter the state of the observed error for all the days of t^{th} month listed in the first column.

c. Count the no of occurrence of each state in t^{th} month. (For eg. say state X_i is occurring j times in the month t of m days, then initial probability of X_i= j/m).

d. Determine the initial probability of all states by using the formulae X_i= J/M where J is the occurrence of i^{th} state in t^{th} month of M days and i= 1, 2, 3.......n..

e. Represent the initial probabilities obtained from step 8 as a row vector (1*n) with n no of entries and is called as initial probability vector denoted by P^o.

6. Construct state occurrence table for t^{th} month and $t+1^{th}$ month.

a. List out all the days of t^{th} and $t+1^{th}$ month in the ascending order as the first column of the table. Assume the number of working days in both months as same.

b. In the second column of the table enter the state corresponding to errors in forecasting for all the days listed in t^{th} month.

c. In column three enter the state corresponding to errors of forecasting for all days listed in the $t+1^{th}$ month.

7. Deduce transition probability matrix from the event occurrence table.

a. Any current state X_i in a particular day of t^{th} month can transform into states X_1, X_2, X_3...........X_n during the same day of $t+1^{th}$ month. Hence there exits n probabilities which results from the probable transformation of current state X_i to other possible states X_1, X_2, X_3...........X_n. Represent these probabilities as P_{11}, P_{12}...........P_{1n}.

b. Form the Transition probability matrix by representing all the current states as rows and next states as columns. Now enter the probabilities as P_{11}, P_{12}...........P_{1n} in 1st row and repeat the same procedure for other rows. Any entry say P_{ij}= No of transformations of current state i of t^{th} month in a particular day to next state j of t+1th month in the same day/ Total no of occurrence of current state i in the tth month.

8. Deduce the current probability vector for the succeeding months t+2, t+3 as

$P^1=P^0*$ TPM

$P^2=P^1*$ TPM

………………

$P^m= P^{m-1}$ *TPM

9. Choose the state with maximum probability from the obtained current probability vector for say the mth month which is a row matrix with probability of each state during say mth month.

10. Determine the possible profit to firm by the adoption of this state of production as indicated by the step 8.

Case study based on indian scenario

The data of sales of a reputed firm was collected for two months and the combined concept of exponential smoothing and Markov based algorithm was applied for it. The firm is selling this item @ Rs 30. Any leftover item is discarded. Cost of each item is Rs16.

Table 1

SL.NO	SALES(OCT)	DISCARDED	PRODN	SALES(NOV)	DISCARDED	PRODN
1	34	11	45	35	12	47
2	37	8	45	38	6	44
3	39	6	45	38	5	43
4	29	16	45	35	11	46
5	36	10	46	37	9	46
6	11	14	25	17	6	23
7	15	10	25	13	9	22
8	37	10	47	33	10	43
9	39	6	45	33	13	46
10	38	11	49	36	11	47
11	34	14	48	36	11	47
12	37	8	45	37	9	46
13	16	8	24	34	10	44
14	12	13	25	22	3	25
15	41	5	46	30	6	36
16	37	11	48	35	8	43
17	39	9	48	34	10	44
18	32	16	48	33	12	45
19	38	12	50	18	6	24
20	15	9	24	19	7	26
21	14	8	22	16	6	22

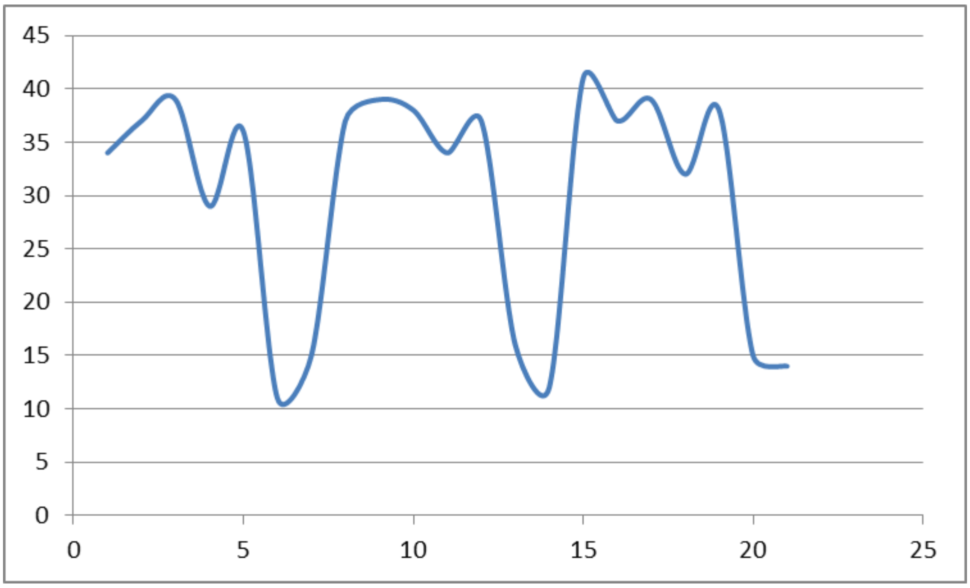
1 Baked product forecast

Table 2

SL.NO	Demand October	Demand November	Forecast October	Forecast October	Error September	Error October
1	34	35	30	30	4	5
2	37	38	31	30	6	8
3	39	38	31	30	8	8
4	29	35	32	30	-3	5
5	36	37	30	31	6	6
6	11	17	31	31	-20	-14
7	15	13	26	31	-11	-18
8	37	33	27	30	10	3
9	39	33	31	29	8	4
10	38	36	32	29	6	7
11	34	36	32	31	2	5
12	37	37	31	32	6	5
13	16	34	31	33	-15	1
14	12	22	27	33	-15	-11
15	41	30	26	31	15	-1
16	37	35	32	31	5	4
17	39	34	31	32	8	2
18	32	33	32	32	0	1
19	38	18	30	32	8	-14
20	15	19	32	29	-17	-10
21	14	16	27	27	-13	-11

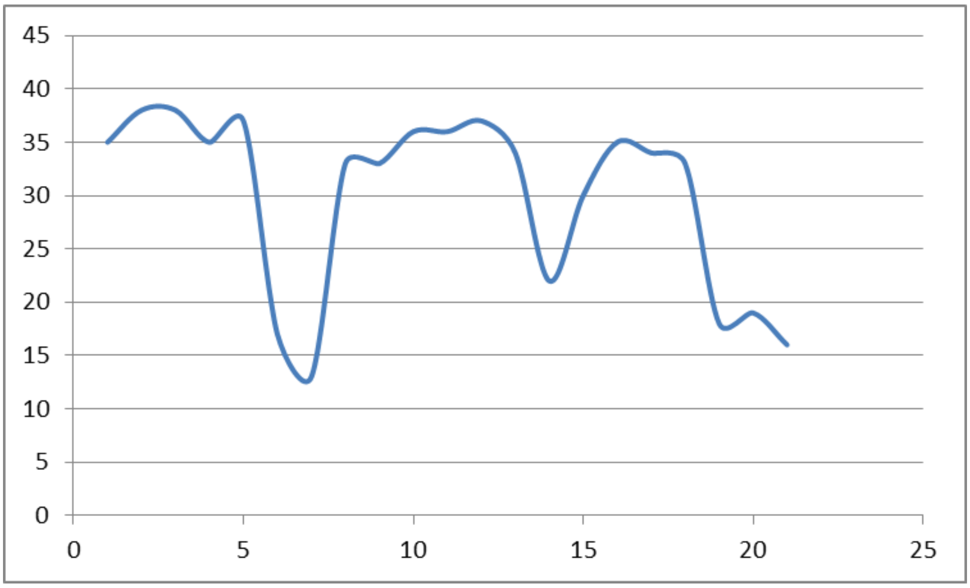
2 Demand plot for october

Figure 1



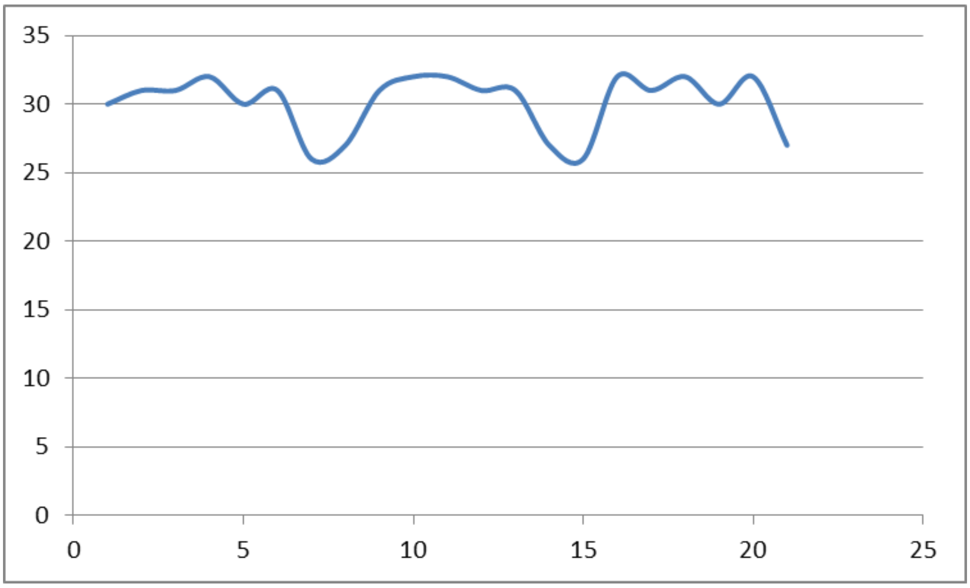
2 Demand plot for november

Figure 2



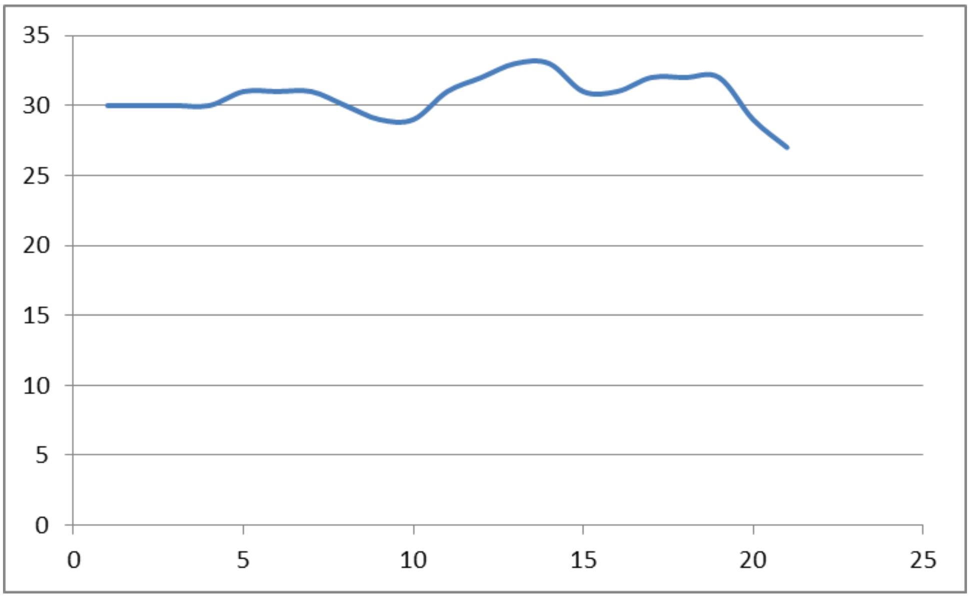
2 Forecast plot for october

Figure 3



2 Forecast plot for november

Figure 4



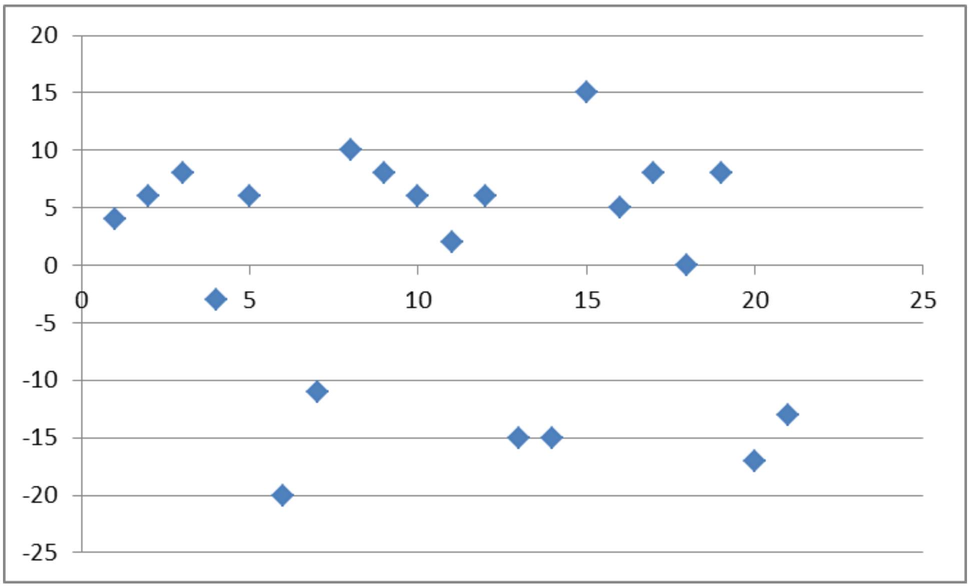
3 Control charts for errors

For constructing a control chart, first determine the MSE. The square root of MSE is used in practice as an estimate of the standard deviation of the distribution of errors. Control charts are based on the assumption that when errors are random, they will be distributed according to a normal distribution around a mean of zero. For a normal distribution, approximately 95.5 percent of the values (errors in this case) can be expected to fall within limits -2S and +2S.

S= (MSE) ^.5
UCL= 0+2S
LCL=0-2S
MSE or Mean squared error = ∑e^2/ (n-1)
Where e is the error and n is the sample size.
For October, S=10.25
2S= 20.5
UCL = 0 + 20.5
LCL = 0 – 20.5

Control chart of errors in demand for october

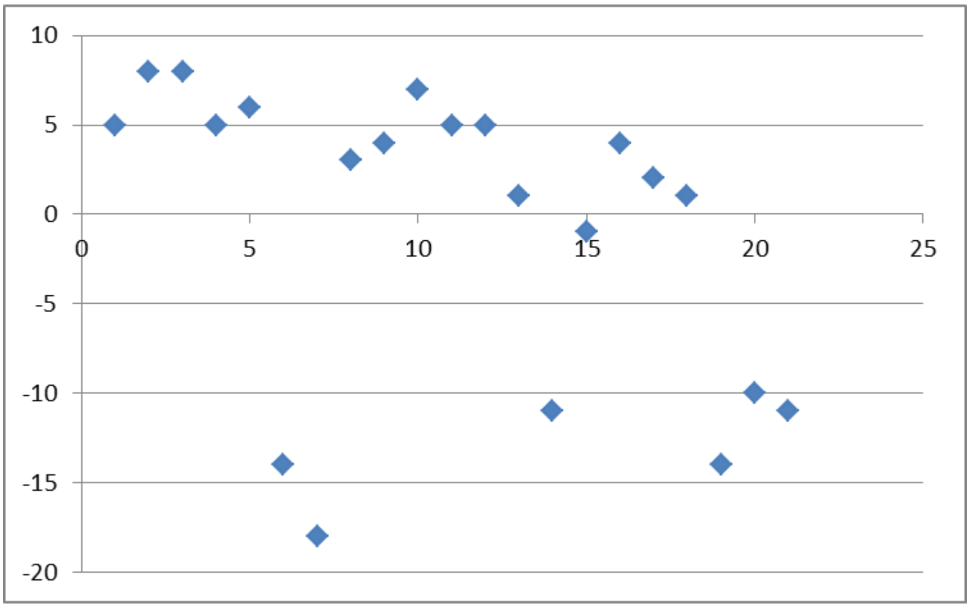
Figure 5



For November, S= 8.42
2S= 16.84
UCL = 0 + 16.84
LCL = 0 – 16.84

Control chart of errors in demand for november

Figure 6



3. Construction of states

The error corresponding to 7th day is out of control and hence the demand of the above date is discarded for the month of November. From the original data the Markov based algorithm is used to predict a single estimate of demand. For this purpose the demand is forecasted by using exponential smoothing method for two consecutive months and error of demand is estimated as the difference between actual demand and forecasted value for each day. Thus we have data pertaining to error of demand for all the days of a month. These errors are divided into equal portions called as states. At first the difference between maximum value of error and minimum value of error is determined giving the range of the errors. In this case the range is 36. Fixing the width as 4, we have 9 states representing the errors and there by the corresponding demand.

4 Deduction of initial probability matrix

Table 3

Class interval of error of demand	State	No of occurrence	Probability
-20,-19,-18,-17	X_1	2	0.1
-16,-15,-14,-13	X_2	3	0.15
-12, -11,-10, -9	X_3	0	0
-8, -7,-6, -5	X_4	0	0
-4-3,-2-1	X_5	1	0.05
0, 1,2, 3	X_6	2	0.1
4,5,6, 7	X_7	6	0.3
8,9,10,11	X_8	5	0.25
12,13,14,15	X_9	1	0.05

The Fourth column of the above table gives initial probability vector P^o for the month October. This matrix gives the initial probability of all states say $X_1, X_2 ... X_{12}$ in the month of October.

P^o = [0.1, 0.15, 0, 0, 0.05, 0.1, 0.3, 0.25, 0.05]

5. Computation of transition probability matrix

State transition table

Table 4

Day	Current state (October)	Subsequent state (November)
1	7	7
2	7	8
3	8	8
4	5	7
5	7	7
6	1	2
8	8	6
9	8	7
10	7	7
11	6	7
12	7	7
13	2	6
14	2	3
15	9	5

16	7	7
17	8	6
18	6	6
19	8	2
20	1	3
21	2	3

Transition Probability Matrix

Table 5

	1	2	3	4	5	6	7	8	9
1	0	0	1	0	0	0	0	0	0
2	0	0	0.67	0	0	0.33	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1	0	0
6	0	0	0	0	0	0.5	0.5	0	0
7	0	0	0	0	0	0	0.833	0.167	0
8	0	0.2	0	0	0	0.4	0.2	0.2	0
9	0	0	0	0	1	0	0	0	0

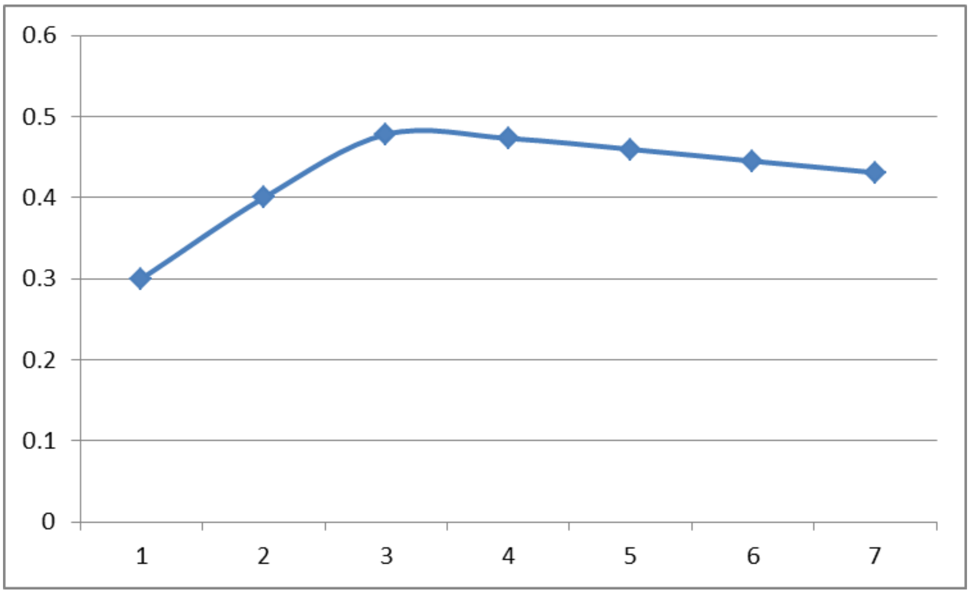
6 Deduction of current probability matrix for suceeding months

Table 6

P^0	0.1	0.15,	0	0	0.05	0.1	0.30	.25	0.05
P^1	0	0.0500	0.2500	0	0.0500	0.1500	0.3999	0.1001	0
P^2	0	0.0200	0.0500	0	0	0.1150	0.4781	0.0868	0
P^3	0	0.0174	0.0200	0	0	0.0922	0.4732	0.0972	0
P^4	0	0.0194	0.0174	0	0	0.0850	0.4597	0.0985	0
P^5	0	0.0193	0.0197	0	0	0.0819	0.4451	0.0965	0
P^6	0	0.0193	0.0205	0	0	0.0795	0.4310	0.0936	0

Probability V/s Time Graph For the state with highest probability

Figure 7



7 Deduction of single point estimate of demand

The maximum probability is for state 7 with errors 4, 5, 6 and 7. The corresponding demands are 34, 36, 37 and 38. To determine the single point estimate of demand, the annual profit by adopting each of the above demand is determined. The demand with maximum profit is selected as the optimum forecast. For this purpose we are introducing the general formulae [S*(P-C)-D*C] per day, where S is the actual demand for a day, D is the discarded items per day (Actual demand-Forecasted demand), C is the cost of making unit quantity, S is the selling price of each product. The demand with maximum savings is chosen as the optimal one.

8 Profits to the firm from existing forecast

Table 7

SL.NO	ACTUAL SALES (S)	FORECASTED DEMAND (F)	DISCARDED ITEMS (D=F-S)	PROFIT (RS) [S*(P-C)-D*C]
1	34	45	11	300
2	37	45	8	390
3	39	45	6	450
4	29	45	16	150
5	36	46	10	344
6	11	25	14	-70
7	15	25	10	50
8	37	47	10	358
9	39	45	6	450
10	38	49	11	356
11	34	48	14	252
12	37	45	8	390
13	16	24	8	96
14	12	25	13	-40
15	41	46	5	494
16	37	48	11	342
17	39	48	9	402
18	32	48	16	192

19	38	50	12	340
20	15	24	9	66
21	14	22	8	68
				RS 5380

The annual profit for the product is Rs 5380*12= Rs 64560

Here since discarded items have positive values there is no under stocking. The under stocking is not considered in further analysis and whenever a negative value appears in the cell of discarded item, it's taken as zero.

9 Profits to the firm when forecast is 34 items per day

Table 8

SL.NO	ACTUAL SALES (S)	FORECASTED DEMAND (F)	DISCARDED ITEMS (D=F-S)	PROFIT (RS) [S*(P-C)-D*C]
1	34	34	0	476
2	37	34	3	470
3	39	34	5	466
4	29	34	-5	406
5	36	34	2	472
6	11	34	-23	154
7	15	34	-19	210
8	37	34	3	470
9	39	34	5	466
10	38	34	4	468
11	34	34	0	476
12	37	34	3	470
13	16	34	-18	224
14	12	34	-22	168
15	41	34	7	462
16	37	34	3	470
17	39	34	5	466
18	32	34	-2	448
19	38	34	4	468
20	15	34	-19	210
21	14	34	-20	196
				8116

The annual profit for the product is Rs 8116*12= Rs 97392

10 Profits to the firm when forecast is 36 items per day

Table 9

SL.NO	ACTUAL SALES (S)	FORECASTED DEMAND (F)	DISCARDED ITEMS (D=F-S)	PROFIT (RS) [S*(P-C)-D*C]
1	34	36	-2	476
2	37	36	1	502
3	39	36	3	498
4	29	36	-7	406
5	36	36	0	504
6	11	36	-25	154
7	15	36	-21	210

8	37	36	1	502
9	39	36	3	498
10	38	36	2	500
11	34	36	-2	476
12	37	36	1	502
13	16	36	-20	224
14	12	36	-24	168
15	41	36	5	494
16	37	36	1	502
17	39	36	3	498
18	32	36	-4	448
19	38	36	2	500
20	15	36	-21	210
21	14	36	-22	196
				8468

The annual profit for the product is Rs 8468*12= Rs 101616

11 Profits to the firm when forecast is 37 items per day

Table 10

SL.NO	ACTUAL SALES (S)	FORECASTED DEMAND (F)	DISCARDED ITEMS (D=F-S)	PROFIT (RS) [S*(P-C)-D*C]
1	34	37	-3	476
2	37	37	0	518
3	39	37	2	514
4	29	37	-8	406
5	36	37	-1	504
6	11	37	-26	154
7	15	37	-22	210
8	37	37	0	518
9	39	37	2	514
10	38	37	1	516
11	34	37	-3	476
12	37	37	0	518
13	16	37	-21	224
14	12	37	-25	168
15	41	37	4	510
16	37	37	0	518
17	39	37	2	514
18	32	37	-5	448
19	38	37	1	516
20	15	37	-22	210
21	14	37	-23	196
				8628

The annual profit for the product is Rs 8628*12= Rs 103536

12 Profits to the firm when forecast is 38 items per day

Table 11

SL.NO	ACTUAL SALES (S)	FORECASTED DEMAND (F)	DISCARDED ITEMS (D=F-S)	PROFIT (RS) [S*(P-C)-D*C]
1	34	38	-4	476
2	37	38	-1	518
3	39	38	1	530
4	29	38	-9	406
5	36	38	-2	504
6	11	38	-27	154
7	15	38	-23	210
8	37	38	-1	518
9	39	38	1	530
10	38	38	0	532
11	34	38	-4	476
12	37	38	-1	534
13	16	38	-22	224
14	12	38	-26	168
15	41	38	3	526
16	37	38	-1	518
17	39	38	1	530
18	32	38	-6	448
19	38	38	0	532
20	15	38	-23	210
21	14	38	-24	196
				8740

The annual profit for the product is Rs 8740*12= Rs 104880

Result

To determine the single point estimate of demand, the annual profit by adopting each of the forecasted demand namely 34, 36, 37 and 38 items are determined. These annual profits are shown below.

Table 12

Forecasted Demand	Annual Profit in Rs
34	97392
36	101616
37	103536
38	104880

The optimal predicted demand with maximum savings is 38 items.

Conclusion

A composite algorithm for determining the demand was developed by incorporating Markov analysis and Exponential smoothing. This algorithm can be used to predict demand of those products with little available time series data. This algorithm takes care of both systematic and random component of demand. The algorithm has been validated by implementing it in a baking firm and by the huge annual savings Rs40320/ product when compared to existing practice. The annual savings can be multiplied by applying it to many similar products. Further this can be modified by incorporating Markov based analysis to other existing statistical methods and estimating the annual savings it can bring to a particular product. These algorithms can be compared on the basis of annual savings that it brings to the firm

References

1. Bijesh and Dr Jayadas A generalized algorithm for the demand prediction of a short life cycle product supply chain and its implementation in a baked product, 2013.
2. Min Huang, Yong He, Haiyan Cen; "Predictive analysis on electric-power supply and demand in China", Renewable Energy 2007; 32(7): 1165–1174.
3. DiyarAkay, Mehmet Atak; "Grey prediction with rolling mechanism for electricity demand forecasting of Turkey", *Energy*; 32(9): 1670-1675, 2007.
4. Boylan, J.E., & Syntetos, A.A. Accuracy and accuracy implication metrics for intermittent demand. Foresight: The International Journal of Applied Forecasting, 4, 39–42, 2006.
5. Armstrong, J.S., & Green, K.C. Demand forecasting: Evidence-based methods. Working paper 24/05. Dept. of Econometrics and Business Statistics, Monash University, 2005.
6. Timmermann, A., & Granger, C. W. J. Efficient market hypothesis and forecasting. International Journal of Forecasting, 20, 15–27, 2004.
7. Allen, P. G., & Fildes, R. Econometric methods. In J. S. Armstrong (Ed.), Principles of forecasting: a handbook for researchers and practitioners (pp. 301–363). New York, Boston, Dordrecht, London, Moscow: Kluwer Academic Publishers, 2001.
8. W. Ching. Markov modulated poisson processes for multi-location inventory problems. Inter. J. Prod. Econ., 53: 232–239, 97. In view of this, Ching et al. proposed a first-order multivariate Markov chain model in [33] for modeling the sales demands of multiple products in a soft drink company.
9. E. Fung W. Ching and M. Ng.A multivariate markov chain model for categorical data sequences and its applications in demand prediction. IMA J. Manag. Math., 13:187 199, 02.
10. [10] E. Fung W. Ching and M. Ng. A higher-order markov model for the newsboy' problem. J. Operat. Res. Soc., 54: 291–298, 03.
11. [11] Zhang SJ, He Y.A Grey–Markov forecasting model for forecasting the total power requirement of agricultural machinery in Shanxi Province. *J Shanxi AgricUniv (Nat Edi)* 21(3): 299–302, 2001.